Measure of Central Tendency.

1. Central Tendency of Data :-

- In many frequency distribution, the tabulated value shows a distinct tendency to cluster or group around a Typical Central value.
- This behaviour of the data about the Concentration of the values in the central part of distribution is called Central Tendency of data.

2. Measure of Central Tendency :-

- There are three Common measure of Central Tendency
  1) Mean
  2) Median
  3) Mode
- The most common and useful measure of central tendency is <u>Mean</u>. A very useful Notation known as Summation Notation.

Summation Notation (Σ) :-

- The symbol Σ read as "Sigma" means Summation.
- If $x_1, x_2, x_3, \ldots, x_n$ be the n values of a variables x, then their Sum $= x_1 + x_2 + x_3 + \ldots + x_n$ is written by using summation notation as $\sum_{i=1}^{n} x_i$ or Simply $\sum x$ or $\sum x_i$

IIIly

The sum $w_1 x_1 + w_2 x_2 + w_3 x_3 + \ldots + w_n x_n$ is denoted by

$$\sum_{i=1}^{n} w_i x_i \quad \text{or} \quad \sum wx \quad \text{or} \quad \sum w_i x_i$$

* Properties of Summation :-

1) $\sum_{i=1}^{n} (x_i \pm y_i) = \sum_{i=1}^{n} x_i \pm \sum_{i=1}^{n} y_i$

2) $\sum_{i=1}^{n} A = A + A + \cdots + A = nA$ (A is Constant)

3) $\sum_{i=1}^{n} A x_i = A x_1 + A x_2 + \cdots + A x_n$

$= A(x_1 + x_2 + x_3 + \cdots + x_n)$

$= A \sum_{i=1}^{n} x_i$

Mean :- Arithmetic Mean ($\bar{x}$)

- There are three types of Mean
  1) Arithmetic Mean (A.M)
  2) Geometric Mean (G.M)
  3) Harmonic Mean (H.M)

Of the three means, Arithmetic Mean is most commonly used.

1) Arithmetic Mean :-

i) Simple Arithmetic Mean :-

- Def^n :- The Arithmetic Mean ($\bar{x}$) of a given series of values say $x_1, x_2, x_3, \ldots, x_n$ is defined as the sum of these values divided by their total number

$\therefore$ Arithmetic mean $(\bar{x}) = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n} = \frac{\Sigma X}{n} = \frac{\Sigma x_i}{n}$

* $\boxed{\text{NOTE}}$:- i) The r^{th} moment about the origin $= \dfrac{\Sigma x^r}{n}$; $r = 1, 2, 3, \ldots$

ii) $\bar{x} = 1^{st}$ moment about the Origin.

Ex. 1 :- Find the Arithmetic mean of 3, 6, 24 & 48

$\Rightarrow$ $$A.M\,(\bar{x}) = \frac{3+6+24+48}{4} = \frac{81}{4} = 20.25$$

2) Weighted Arithmetic Mean :-

- If $x_1, x_2, x_3, \ldots, x_n$ be n values of a variable x and if $f_1, f_2, f_3, \ldots, f_n$ be there respective weights or their respective frequencies then the weighted arithmetic mean is defined by

$$\bar{X} = \frac{f_1x_1 + f_2x_2 + f_3x_3 + \cdots + f_nx_n}{f_1 + f_2 + f_3 + \cdots + f_n} = \frac{\Sigma fx}{\Sigma f} = \frac{\Sigma fx}{N}$$

Where,

$$N = \Sigma f = \text{Total Frequency}.$$

- If the weights $f_1, f_2, \ldots, f_n$ are all equal, then the weighted arithmetic mean becomes a simple arithmetic mean

Ex 2 :- Find the arithmetic mean of the following distribution.

Weight in kg	50	55	60	65	70
Number of men	15	20	25	30	10

$\Rightarrow$

Weight in kg (x)	Number of men (f)	$f \cdot x$
50	15	750
55	20	1100
60	25	1500
65	30	1950
70	10	700
	$N = 100$	$\Sigma fx = 6000$

$\therefore$ Weighted Arithmetic Mean $(\bar{x}) = \frac{\Sigma fx}{N} = \frac{6000}{100} = 60$ kg.

* Important Properties of A.M :-

1. The sum or total of the values is equal to the product of the number of values and their arithmetic mean.
2. The algebraic sum of the deviations of the values from their arithmetic mean
3. Combined Mean :-

If a group having n_1 values has A.M $\bar{x}_1$ and another group having n_2 values has A.M $\bar{x}_2$, then the A.M $(\bar{x})$ of the composite group (i.e. the two groups combined) of $n_1 + n_2$ values is given by

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$\because$ The sum of n_1 observation of the group is $n_1\bar{x}_1$
The sum of n_2 Observation of the group is $n_2\bar{x}_2$

For r groups

$$A.M(\bar{x}) = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + n_3\bar{x}_3 + \cdots + n_r\bar{x}_r}{n_1 + n_2 + n_3 + \cdots + n_r}$$

Ex. 3 :- The means of two samples of sizes 50 & 100 resp. are 54.1 and 50.3 Obtain the mean of the sample size 150 obtained by combining the two samples.

$\Rightarrow$ Given,

$n_1 = 50 \qquad \bar{x}_1 = 54.1$

$n_2 = 100 \qquad \bar{x}_2 = 50.3$

$$\therefore \bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$= \frac{50 \times 54.1 + 100 \times 50.3}{50 + 100}$$

$$\bar{x} = \frac{2705 + 5030}{150}$$

$$\bar{x} = \frac{7735}{150}$$

$$\boxed{\bar{x} = 51.57}$$

* Short-cut Method of Calculating A.M for Discrete Series :-

I] Simple Arithmetic mean :-

– Let $x_1, x_2, x_3, \cdots x_n$ be the n values of variable x and $d_1, d_2, \ldots d_n$ be the deviations of the n values from any arbitrary value A (Assumed mean) then,

$$d_i = x_i - A \qquad ; d_i \text{ – Deviational Constant.}$$

or $$x_i = A + d_i$$

$$\bar{x} = \frac{\sum x_i}{n}$$

$$= \frac{\sum (A + d_i)}{n}$$

$$= \frac{nA + \sum d_i}{n}$$

$$= \frac{nA}{n} + \frac{\sum d_i}{n}$$

$$\boxed{\bar{x} = A + \frac{\sum d_i}{n}}$$

$$A.M = \text{Assumed Mean} + \frac{\text{Sum of the deviations from A}}{\text{Number of values of the variable}}$$

II] Weighted Arithmetic Mean :-

$$\bar{x} = \frac{\sum f_i x_i}{N}$$

$$\bar{x} = \frac{\Sigma f_i(A+d_i)}{N}$$

$$\bar{x} = \frac{\Sigma A f_i + \Sigma f_i d_i}{N}$$

$$= \frac{A \, \Sigma f_i}{N} + \frac{\Sigma f_i d_i}{N} \qquad \left\{ \because \Sigma f_i = N \right\}$$

$$\boxed{A \cdot M(\bar{x}) = A + \frac{\Sigma f_i d_i}{N}}$$

$d_i = x_i - A$

d_i is deviational constant.

Ex. 4 :- The monthly income of 5 persons are in (₹) 250, 360, 280, 480 & 410 find the arithmetic mean of the income of 5 persons

$\Rightarrow$

Income (₹) x	Deviation from 360 $(d = x - 360)$ $(A = 360)$
250	-110
360	0
280	-80
480	120
410	50
Total	$\Sigma d = -20$

Here,

Assumed mean $(A) = 360$, $n = 5$, $\Sigma d = -20$

$$\therefore \bar{x} = A + \frac{\Sigma d}{n}$$

$$= 360 + \frac{-20}{5}$$

$$= 360 - 4$$

$$\boxed{\bar{x} = 356 \text{ ₹}}$$

Ex. 5. Calculate the arithmetic of the following data

Value (x)	1	2	3	4	5	6	7	8	9
frequency (f)	7	11	16	17	26	31	11	1	1.

$\Rightarrow$

Value (x)	frequency (f)	deviations from $A=5$, $d=x-5$	$f \cdot d$
1	7	-4	-28
2	11	-3	-33
3	16	-2	-32
4	17	-1	-17
5	26	0	0
6	31	1	31
7	11	2	22
8	1	3	3
9	1	4	4
Total	$N = 121$	—	$\Sigma fd = -110 + 60 = -50$

Here, $A = 5$, $N = 121$, $\Sigma fd = -50$

$$\therefore \text{Arithmetic mean} = A + \frac{\Sigma fd}{N}$$

$$= 5 + \frac{-50}{121}$$

$$= 5 - 0.41$$

$$\boxed{\bar{x} = 4.59}$$

* Grouped frequency Distribution :-

i) Direct Method :- $\bar{x} = \frac{\Sigma fx}{N}$; N = Total frequency.
x = mid-value of class

ii) Short-cut Method or Assumed Mean :-

$$\bar{x} = A + \frac{\Sigma fd}{N}$$; $d = x - A$
d = (mid-value) − (Assumed value)

iii) Step-Deviation method :-

$$\text{Mean } (\bar{X}) = A + \frac{\Sigma fd}{N} \times i$$

Where,

$$d = \frac{x - A}{i}$$, i = Common width of classes

2. Geometric Mean (G.M) :-

— The geometric mean 'G' of n positive values. Say $x_1, x_2, x_3, \dots, x_n$ is defined by the positive nth root of their product.

$$\therefore \quad G = \sqrt[n]{x_1 \times x_2 \times x_3 \times \dots \times x_n}$$

or

$$G = (x_1 \times x_2 \times x_3 \times \dots \times x_n)^{\frac{1}{n}}$$

3. Harmonic Mean (H.M) :-

– The Harmonic Mean H of the n values $x_1, x_2, x_3, \ldots, x_n$ is defined by

$$\frac{1}{H} = \frac{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \cdots + \frac{1}{x_n}}{n}$$

or

$$H = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \cdots + \frac{1}{x_n}}$$